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FINAL REPORT

For the Period: October 15, 1989 - October 14, 1992.

PERTURBATION PROBLEMS IN FLUID DYNAMICS

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Supported by the Air Force Office of Scientific Research
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TABLE OF CONTENTS

ABSTRACT

Perturbation methods and numerical methods were employed to study five problem areas. (1) For viscous vortical flows, a complete account of the asymptotic analyses, numerical studies and their physical meaning was presented in Springer-Verlag Lecture Notes in 1991. An extension of the asymptotic analysis for the motion and diffusion of a slender vortex filament to allow for the variation of the core structure along the filament was accomplished in 1992. This extension was needed for the study of the vortex breakdown problem. (2) For shock wave interactions, the locations and the types of singularities in the interaction of semilinear waves in three- and higher dimensional space were identified. (3) For wave propagations in a bubbly liquid, a survey of the linear and nonlinear theories and their limitations was presented in an article in Advances in Applied Mechanics in 1991. A system of effective equations uniformly valid at small gas volume fraction and large bubble number density was derived in 1992. (4) For free boundary problems, solutions simulating the breaking up or merging of symmetric slender jets or thin sheets were obtained in 1990. The solution for drop formation after the breaking was formulated recently. (5) In the analysis of structural/acoustic interactions, the solution for the panel oscillation was uncoupled from that for the acoustic field by the recent formulation of the *on surface* conditions taking into account the acoustic effect.

1. INTRODUCTION

We have been using perturbation and numerical methods in our investigations of fluid dynamics problems for many years. Our investigations are centered on five problem areas. They are 1) viscous vortical flows, 2) shock wave interaction 3) multiphase flows, 4) free boundary problems and 5) structural/acoustic interaction. We shall describe briefly our earlier research activities leading to our current investigations in these areas.

The motion and diffusion of slender vortex filaments was analyzed by the method of matched asymptotics in 1965 by Ting and Tung [1]* for two dimensional problems. The same method was employed later to study slender circular vortex rings [2] and vortex filaments [3, 4], in particular to define the velocity of a filament and established the dependence of the velocity on the vortical core structure. These results were needed since the velocity was undefined in the inviscid theory and was unbounded in the limit of zero core size (see for example Lamb [5]). In the analysis of Callegari and Ting [4], both large swirling and axial flows are admitted in the vortical core but the core structure does not vary in the axial direction, i. e., along the filament.

To study the evolution of a distributed vorticity field, i. e., not confined in a slender filament(s), we need numerical solutions of Navier-Stokes (N-S) equations. Numerical simulations of merging of viscous vortices in two-dimensional flows were initiated in 1976 with Lo [6]. To develop an efficient numerical scheme for the solution in a finite computational domain, we need boundary data simulating the solution in an unbounded domain. By making use of the invariants for the moments of vorticity of Truesdell and Moreau (see for example [7]), formulas for the generation of the appropriete boundary data were presented by Ting in 1983 [8]. These formulas were employed to carry out numerical simulations of the interaction and merging of filaments in space (see for example [9]).

In the far field, the velocity induced by an exponentially decaying vorticity field was identified as that of a potential flow induced by doublets, quadrupoles and higher order singularities. The strengths of those singularities were related to the moments of vorticity by Klein and Ting in 1990 [10]. An unified presentation of the theoretical and numerical studies of viscous vortical flows were given in Springer-Verlag Lecture Notes by Ting and Klein in 1991 [11]. There are vortical flows for which the core structure varies along the filament (see for example [12] and [13]), consequently, generalization of the analysis of [4] was made by Klein and Ting in 1992 [14]. Recently studies of the relationships between the moments of vorticity and the meaning

* The number inside square brackets denotes the reference number listed in §4

of the relationships were carried out by Ting and Bauer [16]. Numerical investigations of the motion and merging of vortices were also presented. The research reported in [10], [11], [14] and [15], supported under this grant, will be elaborated in §2.1

The diffraction of weak shocks by structures in a two dimensional space was studied by Ting in 1953 [16] as an extension of the conical solutions of Keller and Blank in 1951 [17] for the diffraction by wedges and corners. The analysis in [17] was later applied to study the wing-body interactions in supersonic flight in 1957 [18] and extended to study the diffraction and reflection of sonic booms by structures in a three-dimensional space in 1968 [19] and in 1977 [20].

The perturbation solution for the diffraction of a shock wave of finite strength by a thin symmetric airfoil of infinite span was obtained by Ting and Ludloff in 1951 [21] as a generalization of Lighthill's conical solutions for the diffraction by thin wedges [22]. The extension to three-dimensional flows was done by Ting and Gunzburger in 1970 [23] to study the diffraction by a moving thin wing at a small angle of attack.

The singular terms in the next order analysis due to the singularites of the derivatives of the leading order solution, (the conical solution) near the sonic circle was studied and removed by Lighthill in 1949 [24] except at a *triple point* where, a weak shock is tangent to the sonic circle. The removal of the singularity at a *triple point* remains as a challenging problem. The singularities produced by the nonlinear interaction of progressing waves in a two-dimensional space were studied by Rauch and Reed in 1982 [25] and by Melrose and Ritter in 1985 [26]. Studies of the singularities of semilinear waves in three and higher dimensional space were carried out under this grant in collaboration with Prof. J. B. Keller at Stanford [27]. The results of the analysis will be presented in §2.2. Study of the singularity near a *triple point* is in progress.

In many engineering problems and problems in nature, the medium is inhomogeneous or multi-phase in a microscopic length scale but the medium behaves as a homogeneous one in a macroscopic length scale, which is much larger than the microscopic scale. The effective equations or the effective coefficients of the equivalent homogeneous medium can often be derived systematically from the basic equations of the phases by the method of multiple scale or the method of homogenization. We have been studying multi-phase flows, in particular the wave propagation in a bubbly liquid, where the gas bubbles are randomly distributed in the liquid. It is an interesting problem because a small gas volume fraction β can change significantly the speed of propagation of the bubbly liquid. Effective equations for wave propagation in bubbly media without viscosity and thermal diffusion were derived by the method of homogenization by Caflisch, Miksis, Papanicolaou and Ting in 1985, [28] and [29] for different ranges of gas volume fraction β for linear and nonlinear theories. The nonlinear theory was derived [28] for the case of very small volume fraction, $\beta = O(N^{-2})$. Here N stands for the bubble number density. In the associated microscopic problem, the dominant mode is the symmetric radial oscillation of bubbles accompanied with volume change. The effective equations are equivalent to those suggested by van Wijngaarden [30]. The linear theory [29] is applicable when the amplitude of oscillation is much smaller than the typical bubble radius and remains valid even for a finite gas volume fraction.

A different nonlinear theory was derived by Miksis and Ting in 1986 [31] for a bubbly media with a gas volume fraction $\beta = O(N^{-1/3})$ which is not so small as the preceding case [28]. In the microscopic problem, the dominant mode oscillation is asymmetric and volume preserving and is coupled with the drift of the bubble center.

The above investigations on bubbly liquids and their extensions to include the thermal and viscous effects were explained in detail in a review article in 1992 [32] by Miksis and Ting. Also under this grant, a composite set of effective equations was derived from the basic equations governing the two phases and their interfaces for $N^{-1} \ll 1$ and $\beta \ll 1$ but is independent of the order of magnitude of β relative to N^{-1} [33]. The highlights of these two articles [32] and [33] will be given in §2.3.

The motion of an interface separating two media is a free surface problem. In general the problem is nonlinear unless the interface is only slightly disturbed. For an interface separating a liquid from a gas phase, e. g., the water wave, the inertia terms in the gas phase can be neglected and hence the pressure on the interface from the gas phase can be considered as a constant. Analytical solutions are available for two dimensional potential flows without surface tension or gravity. Analytical solutions have been obtained for free boundary problems in elasticity and heat tranfer in 1977 [34] and 1983 [35]. An asymptotic method was applied successfully to free surfaces problems, namely, the planing of a flat plate at high Froude number in 1974 by Ting and Keller [36], the buckling of a viscous column by Buckmaster, Nachman and Ting in 1975 [37], Buckmaster and Nachman in 1978 [38] and the surface wave induced by an impinging jet by Miksis and

Ting in 1983 [39]. The drop formation in the breaking of a jet was studied by Taylor in [40] and by Keller [41]. The asymptotic method was applied by Ting and Keller in 1990 [42] to study the similarity solutions of slender jets and thin sheets with surface tension and the connection to drop formation. This result will be enumerated in §2.4. The analysis of the flow field in the drop and its free surface is in progress.

The scattering of weak pulses or acoustic waves by scatterers or inhomogeneous media has been an important practical problem. Asymptotic methods have been employed to study sound propagation through jets in 1980 [43] and the jump conditions across thin bubbly layers with Ng in 1986 [44].

In general, numerical solutions are needed to simulate scattering problems. In order to improve the accuracy of numerical solution of periodic waves in a finite computational domain, higher order radiation conditions were formulated by Engquist and Majda in 1977 [45] and Bayliss and Turkel in 1980 [46]. Exact boundary conditions for the scattering problem were presented by Ting and Miksis in 1986 [47]. In the analysis of the scattering of a pulse by a membrane, it was found by Kriegsmann and Scandrett in 1989 [48] that the solution of the acoustic field can be uncoupled from the solution of the membrane oscillation when the higher order radiation conditions [46] are imposed on the membrane surface. The higher order terms are needed when the incident wave is nearly in resonence with a natural frequency of the membrane and the accuracy of the *on surface* conditions are demonstrated in the comparsion with the numerical solution of the coupled problem. Using asymptotic analysis, higher order *on surface* conditions were derived systematically by Miksis and Ting in 1989 [49] for the two dimensional problem when the acoustic speed is much smaller than the surface wave speed. We have been able to derive systematically the *on surface conditions* to the three dimensional problem of structural/panel interaction [50]. This is an important model problem which simulates the transmission of sound through the fuselage. Description of this recent investigation [50] will be described in §2.5

The investigations supported by this grant were carried out with the collaborations of Dr. Frances Bauer, CIMS and Profs. Egon Krause and Rupert Klein, RWTH Aachen for problem area 1), with Prof. Joseph B. Keller, Stanford University for 2) and 4) and with Prof. Michael J. Miksis, Northwestern University for 3) and 5).

This grant supported the reseach effort of Ting and Bauer at CIMS and two visiting scholars, Prof. S. C. Dai of Shanghai Institute of Applied Mathematics and Mechanics for four months and Dr. Rupert Klein of RWTH Aachen for one and quarter months. A list of publications and presentations of research supported by this grant is given in §3.

2. BRIEF DESCRIPTION OF THE INVESTIGATIONS

The five problem areas of our current research 1) viscous vortical flows, 2) shock wave interaction 3) multiphase flows, 4) free boundary problems and 5) structural/acoustic interactions, are described in the following five subsections §2.1 to §2.5, respectively. The investigations were carried out with the collaborations of Dr. Frances Bauer, CIMS and Profs. Egon Krause and Rupert Klein, RWTH Aachen for problem area 1), with Prof. Joseph B. Keller, Stanford University for 2) and 4) and with Prof. Michael J. Miksis, Northwestern University for 3) and 5).

2.1 Viscous Vortical Flows

For a viscous flow field induced by an initial vorticity distribution of bounded support or decaying exponentially in the far field, the problem in general requires numerical solution of unsteady N-S equations in an unbounded domain. It is then necessary to impose approximate boundary conditions on the finite computational domain which simulate the solution in an unbounded domain and estimate the accuracy of the approximation. Hence, we need the far field behavior of the flow field. In the far field, the vector potential $\mathbf{A}$ for the velocity can be represented by a power series in $|\mathbf{x}|^{-1}$. The coefficient of $|\mathbf{x}|^{-n-1}$ is a homogeneous vector polynomial of the direction cosines of $\mathbf{x}$ of degree n. The coefficients in the polynomial are linear combinations of $3(n+1)(n+2)/2$ nth moments of vorticity. This number is reduced to $J_n = n(n+2)$ on account of Truesdell's consistency conditions on the nth moments [7]. In addition there are the integral invariants of Moreau (see [7]), three for the first and three for the second moments. Using those results, Ting presented the formulas for the approximate boundary data for $\mathbf{A}$, the estimation of their accuracy

and the numerical schemes for the solution of N-S equations [8]. These schemes were employed to study the intersection or merging of vortex filaments with finite cores (See Chapter 3 of [11] and the references therein).

It is known that in the far field, the flow is irrotational and the scalar velocity potential can be expressed as a power series in $|\mathbf{x}|^{-1}$. The coefficient of $|\mathbf{x}|^{-n-1}$ can be represented by $2n+1$ spherical harmonics of nth order. Therefore, only $2n+1$ linear combinations of the J_n vector potentials of nth order in [8] can contribute to far field velocity. These $2n+1$ combinations are identified by Klein and Ting [10, 11] while the remaining n^2-1 combinations are shown to be curl free.

For a viscous flow field induced by an initial vorticity distribution concentrated in a few spots or slender tubular regions in two- or three-dimensional space, we say that the flow field is induced by vortices or vortex filaments and note that the typical core size δ is much smaller than the length scale ℓ of the flow field. The classical inviscid theory is not valid near a point vortex or a straight vortex line, as $\epsilon = \delta/\ell \to 0$, because the velocity becomes unbounded. For a curved vortex line, the velocity of the line is undefined. For a vortex filament of finite core size, the velocity of its center line depends on the core structure in addition to the geometry of the center line (See Lamb [5]). It was recognized by Ting and Tung in 1965 [1], that for a filament with an $O(1)$ total strength or circulation the vorticity in the core has to be $O(\epsilon^{-2})$ and the velocity $O(\epsilon^{-1})$. Hence the viscous terms have to be included in the analysis of the core structure as $\delta/\ell \to 0$. With ϵ as the small parameter, matched asymptotic solutions of N-S equations were constructed so that the velocity field is finite everywhere and the evolution of the core structure and the velocity of the center of the core are defined. The analyses were carried out for the two dimensional case [1], for the axi-symmetric case by Tung and Ting [2] in 1967 and for a vortex filament with only large swirling flow, $O(\epsilon^{-1})$, by Ting [3] in 1971 and with the addition of large axial flow in the core by Callegari and Ting in 1978 [4]. In the three-dimensional analyses, [2] - [4], the core structure does not vary along its centerline $\mathcal{C}$ (no axial variation) and the filament is in the form of a slender torus of order one length $S(t)$, i. e., $S = O(\ell)$.

A comprehensive review of the above numerical and theoretical investigations was given by Ting and Klein in 1991 [11]. In particular, the analysis of Callegari and Ting [4] and its physical meaning and restrictions are described in detail. Two generalizations of the analysis of Callegari and Ting are outlined respectively in subsections **2.3.3.1** and **4.1.2** in [11]. The first one is to allow for axial variation of core structure for a filament of order one length. The second one is to allow for multiple axial length scales, so that the axial variation of the core structure of a long filament, $S \gg \ell$, remains small in the scale ℓ but is order one in a length scale $L \gg \ell$.

Recently Klein and Ting [14] carried out the asymptotic analysis for the first generalization using the same expansion scheme in [4] which assumes only one time scale and two radial scales. For the inner region, i. e., near the centerline $\mathcal{C}$, which is $\mathbf{x} = \mathbf{X}(t,s)$, the Cartesian coordinates are replaced by the radial, circumferential and axial coordinates r, θ, s with $\mathbf{x} = \mathbf{X} + r\hat{r}$ and the radial variable r is then stretched to $\bar{r} = r/\epsilon$. For the velocity relative to the velocity $\dot{\mathbf{X}}$ of $\mathcal{C}$, its circumferential and axial components, v and w, are large, $O(\epsilon^{-1})$ while the radial component remains order one. The leading order governing equations yield

i) ***The leading order core structure, v, w, is symmetric, i. e., independent of θ. But v and w are unknown functions of $t, \bar{r}, s$ to be determined by the compatibility conditions on the higher order equations.***

The higher order equations are linear equations for the higher order solutions, therefore, we can represent the solutions by Fourier series in θ or ϕ and obtain the equations for their Fourier coefficients. The θ-averages of these equations yield the equations for the symmetric parts and the differences of the equations from their θ-averages become the equations for the asymmetric parts.

The asymmetric first order equations, the boundary conditions at $\bar{r} = 0$ and the matching conditions with the outer solutions yield

ii) ***The leading order velocity of the centerline is given by the same set of equations (2.3.50) and (2.3.54) in [11] where the global contribution of the core structure is now a function of t and s.***

The symmetric first order equations are equivalent to the equations for a quasi-steady axi-symmetric inviscid flow and hence yield two compatibility conditions on the core structure,

iii) ***The leading order circulation $\mathcal{G}$ around and the total head $\mathcal{H}$ on a stream tube should remain constant along the tube. The temporal variations of $\mathcal{G}$ and $\mathcal{H}$ are yet to be defined by the compatibility conditions for the symmetric second order equations using the periodicity condition of the solutions on s.***

Thus we obtain a closed system of equations for the motion of the centerline of the filament and the evolution of the core structure. The derivation is presented in [14]. Studies on the numerical solution of this system have been initiated.

Note that the analyses of a slender filament [3], [4] and [14] are based on the assumptions that the flow field has only one time scale and two length scales, δ and ℓ while the length of the filament is $O(\ell)$ and that the reference Reynolds number Re is of the order ϵ^{-2}. Under those assumptions the time-derivative and the viscous term will appear only in the second order equations. The compatibility conditions of the latter yield the evolution equations of the leading order core structure while the initial velocity of C and the initial core structure have to fulfill the compatibility conditions of the first order equations stated in *ii*) and *iii*). When there are problems which do not fit these assumptions we need to introduce multi-scale in the axial variable s and/or in time t. We mention three typical cases requiring multi-time scales:

Case A If the initial velocity of C prescribed is inconsistent with the equations quoted in *ii*) we have to admit a very short time $O(\epsilon^{-2})$ to account for the small effective inertia of the core, $O(\epsilon^2)$. A two-time coupled with two length scales analysis is needed. The analysis was done for the two-dimensional and axi-symmetric problems in [1] and [3]. The analysis for a curved filament is needed.

Case B If the initial core structure does not fulfill the two compatibility conditions in *iii*), we have to admit a short time $O(\epsilon)$ so that unsteady terms will appear in the first order equations.

Case C To study the dynamics of the filament for a long duration, we need a multiple time analysis to account for the slowly varying core size due to diffusion. This was done only for a simple two dimensional case [51].

There are cases requiring multiple length scales in the axial variable s, for example,

Case D There is a slow axial variation of core structure of length scale $L \gg \ell$. The analysis for this case is initiated in **4.1.2** of [11].

Case E There is an intermediate scale between δ and ℓ. See for example the case studied by Klein and Majda [52] and outlined in **4.1.1** of [11].

By combining the above cases we have problems which have multiple scales in time and s in addition to the radial variable r. Also we have to assume different velocity scales and time scales in case that the the circulation is not order one, say $O(\epsilon)$ and/or Re is much smaller than $O(\epsilon^{-2})$, say $O(\epsilon^{-1})$. Consequently, the resultant system of equations will be different. There is an urgent need to study the above cases one by one and identify the relevant physical problems, e. g., the vortex break-down problem (see for example [12], [13]) and the interaction of propeller tip (spiral) vortices. We shall then derive the corresponding closed system of equations and construct the solutions.

During his four month visit at the Courant Institute, S. C. Dai of the Shanghai Institute of Applied Mathematics and Mechanics initiated the study of the the dynamics of three inviscid vortices outside a rigid surface. The study is an extention of the analyses of Synge in 1949 [53] and Tavantzis and Ting in 1988 [54] on the configurations of three inviscid vortices in free space, the steady state solutions and their stability.

Analytical modeling and numerical simulation of the merging of viscous vortices or filaments are reported in Chapter 3 of [11]. For two-dimensional cases, merging of vortices of the same sense, where the total strength $\sum_j \Gamma_j \neq 0$, has been studied extensively. Rules for the final merging of the vortices to a single optimum similarity vortex are formulated. For the special case that the total strength is zero, i. e., in the far field, the flow resembles that induced by an equivalent inviscid doublet.

Studies of the intersection, merging, cancellation and reconnection of vortex filaments are in the early stage. There are a few numerical and experimental results illustrating different types of merging process. Physical understanding, theoretical modeling and numerical simulation of the local merging and reconnection process are needed.

For an exponentially decaying vorticity field, the total strength is zero and hence the far field behaves as a doublet. Thus it is of interest to study the motion and evolution of a viscous doublet, which is a vortical field with total strength zero. Note that the total strength of a two- or three-dimensional viscous doublet is defined by the first moments of vorticity and is a time invariant vector $\mathbf{E}$. In the inviscid theory, stationary point doublets are employed as images to create flows around a closed body. An inviscid theory for the motion of a free point doublet is not available. To begin the investigation of this basic problem of the motion of a viscous doublet we study the long time behavior of the merging of a two-dimensional

viscous vortex pair and of the self-merging of a vortex ring. The definitions of the center of a doublet and its velocity in two-dimensional and axi-symmetric flows are given in [11]. Recently the equations for the center of a three-dimensional viscous vortex and its velocity were formulated [15]. It is found that the long time displacement of the center in two-dimensions differs from that in three-dimensions. In the former the displacement increases as $\ln t$ while in the latter it approaches a stationary point. These analyses and numerical examples will be presented in a forth coming paper by Ting and Bauer [15]. Its title is "Viscous vortices in two- and three-dimensional space" and its abstract is

> Recent developments for the interaction, diffusion and merging of incompressible viscous vortices in two and three dimensional space are reported. We study the motion and evolution of vorticity fields $\Omega(t, \mathbf{x})$ with total strength zero, $<\Omega>= 0$, which is true for a three–dimensional vorticity field decaying exponentially in $|\mathbf{x}|$ and for a two–dimensional case if $<\Omega>= 0$ initially. We called this type of vorticity field a viscous doublet. The strength of the doublet is defined by the time invariant first moments of vorticity. The equations defining the velocity of the center of the doublet in terms of the second moments of vorticity are formulated for two- and three-dimensional problems. We show that the long time behavior of the trajectory of a doublet center in three-dimensional space is different from that in two-dimensional space. Examples are the motion and merging of a slender vortex ring and a two–dimensional vortex pair. For the intersection of two slender filaments (the merging of two segments in a short time interval), a simple model to simulate the merging process and criteria for the reconnection of the filaments after the merging stage are proposed.

2.2 Shock Wave Interaction

In collaboration with Prof. J. B. Keller, Stanford University, we study shock wave interaction in unsteady and/or supersonic flows. We identify the singularities in the interaction of weak shocks and the results are presented in a paper entitled "Singularities in Semilinear Waves" [27] to appear in Comm. Pure Appl. Math. The abstract of the paper is

> New singularities of solutions of *semilinear hyperbolic partial differential equations*, or systems, may be produced when previously existing singularities collide. Our goal is to exhibit the complicated structure of these new singularities for the semilinear wave equation, $(\partial_t^2 - \Delta)u = f(u)$, in N dimensions, $N \geq 2$. We assume that for $t < 0$ the solution u has identical jump discontinuities on $N + 1$ characteristic hyperplanes which are travelling toward the origin and which meet there at $t = 0$. For $t > 0$, these hyperplanes are travelling away from the origin carrying their discontinuities. Inside the expanding simplex bounded by these $N + 1$ hyperplanes there are a number of new singularities. For $N = 2$ Rauch and Reed [25] gave an example of an $f(u)$ for which there is just one new singularity. It is a 5/2 power singularity on the circle inscribed in the triangle bounded by the three lines of jump discontinuities. Then Melrose and Ritter [26] showed that for any smooth $f(u)$ this circle is the only possible locus of new singularities.
>
> We show that for $N = 2$ in general there is a 5/2 power singularity on this circle. In addition, we will exhibit some $f(u)$ for which there is no singularity on it. For $N = 3$ the four planes bearing the initial singularities bound a tetrahedron which shrinks to a point when they collide, and then expands. We shall show that in general the third derivative of u has a logarithmic singularity on the sphere inscribed in the expanding tetrahedron. In addition there are stronger (5/2 power) singularities on the four circular cones from the four vertices tangent to the inscribed sphere. We shall exhibit special $f(u)$ for which these are all the new singularities, and others for which there are no new singularities at all. For any N and general $f(u)$, the solution is singular on the sphere inscribed in the expanding simplex, as well as on certain other conical hypersurfaces. Each of these is a part of the product of a (cone of dimension k) $\times(R^{N-1-k})$, for $k = 2, \cdots, N-1$. The lower the dimension k of the cone, the stronger the singularity.

It is well known that the next order regular perturbation solutions for the diffraction of weak shocks are not valid near the sonic circle or Mach circle. Uniformly valid solutions near the sonic circle can be constructed by the Lighthill technique [24] but the solutions break down near the "triple point" where the plane pulse is tangent to the sonic circle. We note that in the studies of the diffraction of shock waves of finite strengths by weak disturbances by Lighthill [22] and Ting et al [21] [23], the solutions near a sonic circle can be made

uniformly valid by the Lighthill technique [24] and remain valid in the neighborhood of the point where the shock wave cuts off the sonic circle. Recently we began our attempt to incorporate the analyses for weakly disturbed shocks of finite strengths into the solution for the diffraction of weak shocks so that the singularity near the "triple point" can be avoided.

2.3 Wave Propagations in Multiphase Flows

In collaboration with Prof. M. J. Miksis of Northwestern University, we continue our study of wave propagations in bubbly liquids which was initiated in 1984 [28]. Effective equations, or equations in macroscopic scale, for wave propagation in bubbly media without viscosity and thermal diffusion were derived by the method of homogenization by Caflisch, Miksis, Papanicolaou and Ting [28] [29] for different ranges of gas volume fraction for linear and nonlinear theories. The nonlinear theory was derived [28] for the case of very small volume fraction, $\beta = O(N^{-2})$. In the associated microscopic problem, the dominant mode is the symmetric radial oscillation of bubbles accompanied with volume change. A different nonlinear theory was derived by Miksis and Ting [31] for a bubbly media with a gas volume fraction $\beta = O(N^{-1/3})$ which is not so small as the preceding case. In the microscopic problem, the dominant mode oscillation is asymmetric and volume preserving coupled with drift of bubble center. The viscous effects were included in the effective equations later [55].

The canonical microscopic problem for the case of a very small gas volume fraction [28] is that of the radial oscillation of a single bubble driven by external pressure fluctuation. The effect of thermal diffusion across the interface was included by Miksis and Ting [56] and was shown to be of $O(1)$. By using asymptotic analysis the original system of partial differential equations with a free interface was reduced to a system of ordinary integro-differential equations. This reduced system contains three time scales, the short time during resonance, the normal time scale of the forcing period and the long time scale due to the slow diffusion process. An efficient numerical scheme for this system and numerical results showing the long time accumulative effect of diffusion were presented in [57].

The above investigations on bubbly liquids were explained in detail in a review article [32] by Miksis and Ting in 1992. Also in the review article [32] a composite set of effective equations was proposed. This composite set was assumed to be valid for both ranges of β, i. e., independent of its relationship to N^{-1}. This composite set was derived from the basic equations governing the two phases and their interfaces for $\beta \ll 1$ in addition to $N^{-1} \ll 1$ in a paper [33] in 1992 by Miksis and Ting. Thus the preceding proposition in [32] is verified. The abstract of the paper [33] is :

> A system of effective equations for wave propagation in a bubbly liquid with a small gas volume fraction is obtained. The derivation consists of averaging the basic equations of the two-phase flow and then using the method of matched asymptotic expansions in order to define the microscopic problem. The resulting system consists of an effective continuity equation with a source term and momentum equations with forcing terms. These terms are coupled to a canonical microscopic problem for the motion of a single bubble. The system accounts for the nonlinear effects of the translations of the bubble centers and the oscillations of the bubble interfaces including the spherically symmetric mode and all the asymmetric modes. The system remains valid independent of the order of magnitude of the gas volume fraction relative to the reciprocal of the bubble number density.

Many problems related to nonlinear wave propagation in bubbly media remain to be analyzed. In particular, the properties of the effective equations derived in [33] and their solutions have not been studied carefully. Also the solutions of the associated canonical microscopic problems remain to be constructed. These statements apply equally well to the set of effective equations derived in [31] for a bubbly media with a not too small gas volume fraction.

2.4 Free Boundary Problems

In collaboration with Keller, the mass flux variation in an axi-symmetric slender jet or sheet and the motion of the free surface with surface tension was studied [42]. The solution can be used to model the break up or merging of jets or thin sheets. The analysis was published. The abstract of the paper [42] is

Simplified equations governing the potential flow and the shape of the slender jets and thin sheet of liquid are derived, taking into account surface tension. Families of similarity solutions of these equations are introduced. For jets and for symmetric sheets they satisfy ordinary differential equations. The properties of these similarity solutions are examined analytically and numerically. They can be used to describe the motion of a liquid sheet on a solid, the thickening and flow following the breaking of jets, the merging of two jets, the formation or closing of holes or slits in sheets, etc. Some applications to such problems are given.

We are now studying the drop formation after the breaking and the flow in the neck connecting the drop to the jet.

We are generalizing the analysis to a slender jet in space, i. e., to study the motion of of the jet surface and its centerline. The slenderness ratio ϵ is employed as the expansion parameter. The the leading order solutions give the variations of the axial mass flux vector and the cross sectional area in a quasi one-dimensional flow and the motion of the centerline. We shall examine one by one the effects of the viscosity, surface tension and gravity on the leading order equations and their solutions by changing the orders of magnitude of the Reynolds number, Weber number and/or Froude number relative to ϵ. Also we shall derive the next order equations and try to identify them as equations for the moment of momentum over a cross-section area. For example, when the mass flux is of higher order the next order equations become the leading ones, which can be identified as the equations employed by Buckmaster and Nachman to study the buckling and stretching of a viscida in two-dimensional space [37, 38]. Analysis for the buckling problem in three-dimensional space has been initiated.

2.5 Structural/Acoustic Interaction

The scattering of an incident pulse from an interface separating two fluids in two-dimensional space was studied by Miksis and Ting in 1989 [49]. The interface can be either a plain interface with surface tension or a membrane. We have completed a generalization of the above analysis [49] to a three dimensional problem, the transmission and reflection of acoustic waves by a flexible panel. In the limit where the ratio of the acoustic wave length to surface wave length is small, we derive a condition on the panel, a *on surface* condition, which accounts for the interaction of the acoustic wave and the panel oscillation. With this condition, the solution of the panel oscillation is uncoupled from that of the acoustic field. The solution of the panel oscillation in turn defines the transmitted and reflected waves. The investigation will be reported in a paper entilted "On surface condition in acoustic wave and flexible panel interaction" [50]. The proposed abstract is:

The scattering of an incident wave by a flexible panel with fixed edges serves as a model problem to study the transmission of incident waves through an airframe. The panel oscillation $\eta(t, x, z)$ is governed by the partial differential equation for a vibrating plate while the loading, the pressure difference across the plate, depends on the incident and reflected waves above the panel, $y > 0$, and the transmitted wave below, $y < 0$. Thus the solution of the plate equation is coupled with the solutions of the wave equation in the half space above and below the panel subject to the far field radiation condition. We express the loading as a double integral of the panel oscillation in retarded time. Thus we have an *exact on surface* condition. The interaction problem is then governed by an integro-differential equation for $\eta(t, x, z)$ over the panel. When the acoustic wave length is much smaller than the size of the panel, we employ the method of asymptotic expansion to derive the first and second order approximations to the double integral for the loading and obtain the first order and second order *on surface* conditions. The integro-differential equation for η is then reduced to partial differential equations with the first and second order corrections. Earlier studies of the interaction of acoustic waves with a membrane showed that the second order correction is needed when the incident wave is nearly in resonance with a natural frequency of the panel. Numerical solutions of the acoustic/panel interaction using these approximate on surface conditions are constructed and compared with the solution using the *exact* condition to establish the accuracy of these approximate *on surface* conditions.

We intend to extend this analysis [50] to account for the effects of semi-linear surface waves when the amplitude of the plate oscillation is not very small relative to the plate thickness and the effects of a flow

field on one side of the surface. Thus the solution can be used to simulate the noise transmitted inside a fuselage due to the coupling of the noise in the external flow with the fuselage panel oscillation.

3. PUBLICATIONS AND PRESENTATIONS SUPPORTED BY THIS GRANT

3.1 Publications

(1) Klein, R. and Ting, L., "Far field velocity potential induced by a rapidly decaying vorticity distribution", ZAMP, Vol.41, pp 395-418, 1990.

(2) Ting, L. and Keller, J. B., " Slender Jets and Thin Sheets With Surface Tension", SIAM J. Appl. Math., Vol. 50, pp 1533-1546, 1990.

(3) Ting, L. and Klein, R., *Viscous Vortical Flow*, Springer-Verlag Lecture Notes in Physics No. 374, 1991.

(4) Miksis, J. B. and Ting, L., "Effective Equations for Multiphase Flows - Waves in Bubbly Liquid", Advances in Applied Mechanics, Vol. 28, Eds. J. Hutchinsoı and T. Y. Wu, Academic Press, 141-260, 1991.

(5) Klein, R. and Ting, L. "Vortex filament with axial core structure variation", Appl. Math. Letters, Vol. 5, 99-103, 1992.

(6) Miksis, M. J. and Ting, L. "Wave Propagation in a Bubbly Liquid at Small Volume Fraction", Comm. Chem. Engrg, Vol. 118, 59-73, 1992.

(7) Keller, J. B. and Ting, L., "Singularities of Semilinear Waves", to appear in Comm. Pure Appl. Math., Vol. XLVI, 1-12, 1993.

(8) Ting, L. and Bauer, F., "Viscous Vortices in Two- and Three-Dimensional Space", submitted to J. Compu.& Fluids on October 22, 1992.

(9) Miksis, M. J. and Ting, L., "On Surface Condition in Acoustic Wave and Flexible Panel Interaction", in preparation, to be sumbitted to Appl. Math. Letters.

3.2 *Invited Talks*

1) L. Ting, "Interaction of a Shock Wave and Interface Separating Two Fluids", Minisymposium: Free Boundary Problems in Fluid Dynamics, 1990 SIAM Annual meeting in Chicago, July 16-20.

2) F. Bauer and L. Ting, "Interaction of Vortex Filaments, Intersection, Cancellation and Reconnection", Minisymposium M52, *Dynamics of Slender Vortices*, Second International Conference on Industrial and Applied Mathematics, July 8-12, 1991, Washington, DC.

3) L. Ting, "Viscous Vortical Flows", Session E2, Symposium of the Division of Fluid Dynamics, the March Meeting of the American Physical Society, 16-20 March 1992, Indianapolis, IN.

4) L. Ting, "Recent Developments in viscous vortical flow", Lectures in Applied Mathematics, Northwestern University, April 30, 1992.

4. REFERENCES

[1] Ting, L. and Tung, C., "On the Motion and Decay of a Vortex in a Non-Uniform Stream", Physics of Fluids, Vol. 8, 1039-1051, 1965.

[2] Tung, C. and Ting, L., "Motion and Decay of a Vortex Ring", Physics of Fluids, Vol. 10, 901-910, 1967.

[3] Ting, L., "On the Motion and Decay of Vortices", 11-40, of *Aircraft Wake Turbulence and Detections*, edited by J.R. Olsen, A. Goldburg and M. Rogers, Plenum Press, NY 1971.

[4] Callegari, A. and Ting, L., "Motion of a Curved Vortex Filament with Decaying Vortical Core and Axial Velocity", SIAM Journal on Appl. Math., Vol. 15, 148-175, 1978.

[5] Lamb, H., *Hydrodynamics*, Dover Publ., New York, 1932.

[6] Lo, K.C.R. and Ting, L., "Studies of Merging of Vortices," Phys. of Fluids, Vol. 19, 912-913, 1976.

[7] Truesdell, C., *The Kinematics of Vorticity*, Indiana University Press, 1954.

[8] Ting, L., "On the Applications of Integral Invariants and Decay Laws of Vorticity Distributions," J. Fluid Mech., Vol. 127, 497-506, 1983.

[9] Chamberlain, J. P. and Liu, C. H., "Navier-Stokes Calculations for Three-Dimensional Flows in Unbounded Domain", AIAA J., Vol. 23, 868-874, 1985.

[10] Klein, R. and Ting, L., "Far field velocity potential induced by a rapidly decaying vorticity distribution" ZAMP, Vol. 41, 395-418, 1990.
[11] Ting, L. and Klein, R., *Viscous Vortical Flow*, Springer-Verlag Lecture Notes in Physics No. 374, 1991.
[12] Krause, E., "Investigations of Vortex Breakdown by Numerical Solution of N-S Equations", in *Second International Colloquium on Vortical Flows, Vortex Control and Breakdown Behavior*, pp.43-57, Brown Boveri Research Centre, Baden, Switzerland, April 6-7, 1987.
[13] Reyna, L., Menne, S., "Numerical Prediction of Flow in Slender Vortices", Comp. & Fluids, Vol. 16, 239-256, 1988.
[14] Klein, R. and Ting, L. "Vortex filament with axial core structure variation", Appl. Math. Letters, Vol. 5, 99-103, 1992.
[15] Ting, L. and Bauer, F., "Viscous Vortices in Two and Three dimensional Space", submitted to J. Computers & Fluids, October 22, 1992.
[16] Ting, L., "Diffraction and Reflection of Weak Shocks by Structures," J. Math. and Phys., Vol. XXXII, 102-116, 1953.
[17] Keller, J. B. and Blank, A., "Diffraction and Reflection of Pulses by Wedges and Corners", Comm. Pure Appl. Math., Vol.4, 75-94, 1951.
[18] Ting, L., "Diffraction of Disturbances Around a Convex Right Corner with Application in Acoustics and Wing-Body Interference," J. Aeron. Sc., Vol. 24, 821-830, 1957.
[19] Ting, L. and Pan, Y.S., "Incidence of N-Waves on Structures," Second Conference on Sonic Boom Research, NASA SP-180, May 1968.
[20] Ferri, A., Ting, L. and Lo, R., "Non-linear Sonic Boom Propagation Including the Asymmetric Effects," AIAA J., Vol. 15, 653-658, 1977.
[21] Ting, L. and Ludloff, H.F., "Aerodynamics of Blasts," J. Aeron. Sc., Vol. 19, 317-328, 1952.
[22] Lighthill, M. J., "The Diffraction of Blast - I, -II", Proc. Royal Soc., A, **198**, **200**, 1949, 1950.
[23] Ting, L. and Gunzburger, M., "Diffraction of Shock Wave by Moving Thin Wing", J. Fluid Mechanics, **42**, 585-608, 1970.
[24] Lighthill, M. J., " The Shock Strength in Supersonic Conical Flow ", Phil. Mag., Series 7, Vol. 40, 1201-1221, 1949.
[25] Rauch, J. B. and Reed, M. C., "Singularities produced by the nonlinear interaction of three progressing waves: examples", Comm. Partial Diff. Eqs., Vol. 7, 1117–1133, 1982.
[26] Melrose, R. B. and Ritter, N., "Interaction of progressing waves for semilinear wave equations. I.", Ann. Math., Vol. 2, 187–213, 1985.
[27] Keller, J. B. and Ting, L., "Singularities of Semilinear Waves", to appear in Comm. Pure Appl. Math., Vol. XLVI, 1-12, 1993.
[28] Caflisch, R.E., Miksis, M.J., Papanicolaou, G.C., and Ting, L., "Effective Equations for Wave Propagation in Bubbly Liquids", J. Fluid Mech., Vol. 153 , 959-973, 1985.
[29] Caflisch, R.E., Miksis, M.J., Papanicolaou, G.C. and Ting, L., "Wave Propagation in Bubbly Liquids at Finite Volume Fraction", J. Fluid Mech., Vol. 160, 1-14, 1985.
[30] van Wijngaarden, L., "On the Equations of Motion for Mixtures of Liquid and Gas Bubbles", J. Fluid Mech., Vol. 4, 465-474, 1968.
[31] Miksis, M.J. and Ting, L., "Wave Propagation in a Bubbly Liquid with Finite Amplitude Asymmetric Oscillations", Phys. Fluids, Vol. 29, 603-618, 1986.
[32] Miksis, J. B. and Ting, L., " Effective Equations for Multiphase Flows - Waves in Bubbly Liquid"", Advances in Applied Mechanics, Vol. 28, Eds. J. Hutchinson and T. Y. Wu, Academic Press, 141-260, 1991.
[33] Miksis, M. J. and Ting, L. "Wave Propagation in a Bubbly Liquid at Small Volume Fraction", Comm. Chem. Engrg, Vol. 118, 59-73, 1992.
[34] Ting, L., "Interaction of Two Vortices in a Fluid with Lower Stagnation Pressure," Quarterly of Applied Math., Vol. 34, 353-363, 1977.
[35] Ting, L., Ahluwalia, D.S. and Miksis, M.J., "Solutions of a Class of Mixed Free Boundary Problems," SIAM J. Appl. Math., Vol. 43, 759-775, 1983.
[36] Ting, L. and Keller, J.B., "Planning of a Flat Plate at High Froude Number," Phys. of Fluids, Vol. 17, 1974, pp. 1080-1086, 1974.

[37] Buckmaster, J. D., Nachman A. and Ting, L., The Buckling and Stretching of a Viscida", J. Fluid Mech., Vol. 69, 1-20, 1975.
[38] Buckmaster, J. D. and Nachman, A., "The Buckling and Stretching of a Viscida II. Effects of Surface Tension", Quart. J. Mech. Appl. Math., Vol. XXXI, 157-168, 1978.
[39] Miksis, M.J. and Ting, L., "Surface Waves Induced by an Impinging Jet," Physics of Fluids, Vol. 26, 2378-2384, 1983.
[40] Taylor, G. I., "The Dynamics of Thin Sheets of Fluids, I. Water Bells, II. Waves on Liquid Sheets, III. Disintegration of Fluid Sheets", Proc. Roy. Soc. London, Ser. A, Vol 253, 289-295, 296-312, 313-321, 1959.
[41] Keller, J. B., "Breaking of Liquid Films and Threads", Phys. Fluids, Vol. 26, 3451-3453, 1983.
[42] Ting, L. and Keller, J. B., "Slender Jets and Thin Sheets With Surface Tension", SIAM J. Appl. Math., Vol. 50, 1533-1546, Dec. 1990.
[43] Ting, L., "Sound Propagation through Parallel Jets Exiting from Ducts", J. Acoust. Soc. Amer., Vol. 67, 782-791, 1980.
[44] Ng, K. C. and Ting, L. , "Wave Propagation through a Thin Bubbly Layer", J. Acoust. Soc. Am., Vol. 79, 924-926, 1986.
[45] Engquist, B. and Majda, A., "Absorbing Boundary conditions for the Numerical Simulation of Waves", Math. Comp., Vol. 31, 629-652, 1977.
[46] Bayliss, A. and Turkel, E., "Radiation Boundary Conditions for Wave Like Equations", Comm. Pure Appl. Math., Vol. 23, 707-725, 1980.
[47] Ting, L. and Miksis, M.J., "Exact Boundary Conditions for Scattering Problems", J. Acoust. Soc. Amer., Vol. 80, 1825-1827, 1986.
[48] Kriegsmann, G. A. and Scandrett, C. L., "Assessment of a new radiation damping model for structure acoustic interactions", J. Acoust. Soc. Amer., Vol. 86, 788-794, 1989.
[49] Miksis, M.J. and Ting, L., "Scattering of an Incident Wave from an Interface Separating Two Fluids", Wave Motion., Vol. 11, 545-557, 1989.
[50] Miksis, M. J. and Ting, L., "On Surface Condition in Acoustic Wave and Flexible Panel Interaction", in preparation.
[51] Gunzburger, M. D., "Long Time Behavior of a Decaying Vortex", ZaMM, Vol. 53, 751-760, 1973.
[52] Klein, R. and Majda, A. J., "Self-Stretching of a perturbed Vortex Filament I: The Asymptotic Equation for deviations from a Straight Line", Physica D, Vol. 49, 323-352, 1991.
[53] Synge, J. L., "On the Motion of Three Vortices", Can. J. Math., Vol. 1, 257-270, 1949.
[54] Tavantzis, J and Ting, L., "The Dynamics of Three Vortices Revisited", Phys. Fluids, Vol. 31, 1392-1409, 1988.
[55] Miksis, M.J. and Ting, L., "Viscous Effects on Wave Propagation in a Bubbly Liquid", Phys. Fluids, **30**, pp. 1683-1689, 1987.
[56] Miksis, M.J. and Ting, L., "Nonlinear Radial Oscillations of a Gas Bubble Including Thermal Effects", J. Amer. Acoust. Soc., **76**, pp. 897-905, 1984.
[57] Miksis, M. and Ting, L., "A Numerical Study of Thermal Effects on Nonlinear Bubble Oscillations", J. Acoust. Soc. Am., **81**, pp. 1331-1340, 1987.